

# Extraction of two-photon contributions to the proton form factors

J. Arrington

*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

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Significant discrepancies have been observed between proton form factors as measured by Rosenbluth separation and polarization transfer techniques. There are indications that this difference may be caused by corrections to the one photon exchange approximation that are not taken into account in standard radiative correction procedures. In this paper, we constrain the two-photon amplitudes by combining data from Rosenbluth, polarization transfer, and positron-proton scattering measurements. This allows a rough extraction of these two-photon effects in elastic electron-proton scattering, and provides an improved extraction of the proton electromagnetic form factors.

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## INTRODUCTION

Extractions of the proton electromagnetic form factors utilizing the polarization transfer technique show a significant decrease in the ratio of electric to magnetic form factor at large momentum transfers [1, 2]. These results contradict a large body of Rosenbluth separation measurements [3, 4, 5] which indicate approximate scaling of the form factors,  $\mu_p G_E/G_M \approx 1$ . This inconsistency leads to a large uncertainty in our knowledge of the proton electromagnetic form factors and could have significant implications for other experiments which rely on similar techniques or which assume knowledge of the proton form factors to interpret their data [6, 7, 8].

It has been suggested that two-photon exchange contributions could be responsible for the discrepancy between the Rosenbluth, or Longitudinal-Transverse (L-T), separation and polarization transfer form factors [9]. Calculations of the two-photon exchange diagram suggest that this may indeed be the case [10, 11], and there is some evidence for two-photon exchange in comparisons of electron-proton and positron-proton scattering [12]. However, a complete calculation of two-photon exchange must include contributions where the intermediate proton is in an excited state, which are not included in Ref. [10]. In Ref. [11], the contribution from intermediate states is included through two-photon scattering off of partons in the proton, with emission and re-absorption of the partons by the nucleon described in terms of generalized parton distributions. However, this approach is not expected to be valid at low four-momentum transfers,  $Q^2$ , or for small values of the virtual photon polarization,  $\varepsilon$ , and yields approximately half of the effect needed to bring the two techniques into agreement at larger  $Q^2$ .

A general formalism does exist for parameterizing contributions beyond the one-photon (Born) approximations [9]. While discussed in terms of two-photon contributions, this formalism includes all terms in the elastic scattering amplitude: vertex corrections, loop corrections (vacuum polarization), soft and hard two-photon contributions, and multi-photon exchange; all terms with just

the electron and proton in the final state. In the Born approximation, one obtains two real amplitudes which depend only on the momentum transfer:  $G_E(Q^2)$  and  $G_M(Q^2)$ . In the generalized case, there are three complex amplitudes which depend on both  $Q^2$  and  $\varepsilon$ :  $\tilde{G}_E(\varepsilon, Q^2)$ ,  $\tilde{G}_M(\varepsilon, Q^2)$ , and  $\tilde{F}_3(\varepsilon, Q^2)$ . For convenience, we break up the generalized form factors into the Born values and the “two-photon” contributions, e.g.  $\tilde{G}_E(\varepsilon, Q^2) = G_E(Q^2) + \Delta G_E(\varepsilon, Q^2)$ , and define  $Y_{2\gamma}$ ,

$$Y_{2\gamma} = \mathcal{R}e \left( \frac{\nu \tilde{F}_3}{M_p^2 |G_M|} \right), \quad (1)$$

where  $\nu = M_p^2 \sqrt{(1+\varepsilon)/(1-\varepsilon)} \sqrt{\tau(1+\tau)}$  (equivalent to the definition given in Ref. [9]). We now have the two usual Born-level form factors and three two-photon amplitudes:  $\Delta G_E$ ,  $\Delta G_M$ , and  $Y_{2\gamma}$ . The first two are complex, but as long as they are not too large, only the real portion of these amplitudes has any significant effect on the observables discussed below, so throughout this paper we will refer only to the real part of  $\tilde{G}_E$  and  $\tilde{G}_M$ .

The goal of this work is to use the existing data on elastic electron-proton and positron-proton scattering data to estimate the small two-photon amplitudes, and then use these amplitudes to correct the form factors extracted from polarization transfer and Rosenbluth separation measurements.

## EXTRACTION OF THE TWO-PHOTON AMPLITUDES

The proton form factor ratio,  $G_E/G_M$ , has been extracted from cross section and polarization transfer measurements assuming one-photon exchange. In the generalized formalism, the extracted ratio does not yield the true form factor ratio, but is a function of these generalized form factors:

$$R_{Pol} = (\tilde{G}_E/\tilde{G}_M) + (1 - \frac{2\varepsilon}{1+\varepsilon} \tilde{G}_E/\tilde{G}_M) Y_{2\gamma}, \quad (2)$$

$$R_{L-T}^2 = (\tilde{G}_E/\tilde{G}_M)^2 + 2(\tau + \tilde{G}_E/\tilde{G}_M) Y_{2\gamma}, \quad (3)$$

where  $\tau = Q^2/4M_p^2$ . Keeping terms up to order  $\alpha_{EM}$ , the change to the reduced cross section ( $\sigma_r = \tau G_M^2 + \varepsilon G_E^2$  in the Born approximation) is

$$\frac{\Delta\sigma_r}{G_M^2} \approx 2\tau \frac{\Delta G_M}{G_M} + 2\varepsilon \rho^2 \frac{\Delta G_E}{G_E} + 2\varepsilon(\tau + \rho)Y_{2\gamma} \quad (4)$$

where  $\rho = G_E/G_M$ .

The general procedure for extracting the two photon amplitudes is as follows: From Eqs. 2 and 3, we can see that it is only the  $Y_{2\gamma}$  term that leads to a difference between the polarization transfer and L-T form factor ratio, and so this difference will allow us to determine  $Y_{2\gamma}$ . To obtain the true (Born) form factors we must still determine  $\Delta G_M$  and  $\Delta G_E$ . Because the dominant terms of the two-photon correction changes sign for positron-proton scattering, we can use the existing data for positron-proton scattering as an additional constraint on  $\Delta G_E$  and  $\Delta G_M$ , allowing an extraction of the true form factors,  $G_E$  and  $G_M$ , corrected for two-photon (and multi-photon) exchange contributions. These are the form factors that can be directly connected to the structure of the proton, and which can be compared to lattice calculations or models of the nucleon.

Given the limitations of the existing cross section, polarization transfer, and positron-proton scattering data, we are forced to make some assumptions in the extraction of the two-photon amplitudes. First, we assume that two-photon effects are responsible for all of the discrepancy. Second, we assume that the two-photon amplitudes depend weakly on  $\varepsilon$ , although we will examine the effect of  $\varepsilon$ -dependence in the error analysis. Finally, we only consider processes that are of order  $\alpha_{EM}$  with respect to the born amplitudes, and neglect higher order corrections. Specifically, we neglect terms other than the “standard” radiative corrections, two-photon exchange, and soft multi-photon exchange (“Coulomb distortion”) which is  $\mathcal{O}(\alpha_{EM})$  after resummation. With these assumptions, it is possible to constrain the “two-photon” contributions to the form factors well enough to extract  $G_E$  and  $G_M$ , albeit with additional uncertainty due to these two-photon corrections. Further data would allow improved extractions of the two-photon amplitudes, as well as provide better tests of the assumptions used in the analysis.

In the analysis of the existing cross section data, two-photon exchange contributions were only treated approximately, while other radiative corrections of  $\mathcal{O}(\alpha_{EM})$  were applied. However, except for the well understood Bremsstrahlung corrections, all of these terms are included in the two-photon amplitudes  $\Delta G_E$  and  $\Delta G_M$ . In principle, the corrections that were applied to the measured cross sections should be removed before using the data to extract these amplitudes. However, it turns out that this is not necessary. The extraction of  $Y_{2\gamma}$  comes from the difference between  $R_{Pol}$  and  $R_{L-T}$ , but  $R_{Pol}$  is insensitive to these radiative corrections, while  $R_{L-T}$

depends only on the  $\varepsilon$ -dependent corrections to the cross section. Because the loop and vertex corrections are independent of  $\varepsilon$ , they do not modify the value of  $R_{L-T}$  extracted from the data, and so do not change the extracted value of  $Y_{2\gamma}$ .

The other two-photon amplitudes will be constrained by the comparison of electron-proton and positron-proton charge sign with the charge of the lepton are extracted: the two-photon exchange and the Coulomb corrections. Because the loop and vertex corrections are identical for positron and electron scattering, their inclusion does not modify the comparison of positron and electron data. Strictly speaking, the contributions due to loop and vertex diagrams are not *corrections* to the generalized form factors, they are included in  $\Delta G_M$  and  $\Delta G_E$ . However, the goal is to obtain the true form factors and applying the loop and vertex to the measured cross sections yields the same result as including them in the two-photon corrections  $\Delta G_E$  and  $\Delta G_M$ . Similarly, while soft multi-photon exchange (Coulomb distortion) can play a non-negligible role at low-to-moderate  $Q^2$  values [13], these corrections should not be applied to the data for this analysis, as they are included as part of the higher-order corrections ( $\Delta G_M$ ,  $\Delta G_E$ , and  $Y_{2\gamma}$ ).

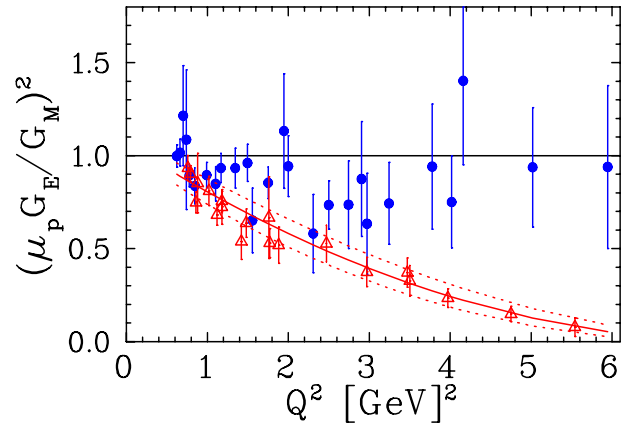


FIG. 1: (Color online) Rosenbluth form factor ratio squared,  $R_{L-T}^2$  (blue circles), polarization transfer ratio squared,  $R_{Pol}^2$  (red triangles), and the parameterization of the polarization transfer ratio and uncertainty (solid and dotted red lines) used in the extraction of  $Y_{2\gamma}$ . The error bars shown on the Rosenbluth extractions include an estimate of the uncertainty in the determination of the normalization of the different data sets in the global analysis.

To extract  $Y_{2\gamma}$ , we compare the Rosenbluth extraction of  $G_E/G_M$  from a global analysis of cross section data (Fig. 2 of Ref. [6]) to a parameterization of the polarization transfer results and uncertainties, as shown by solid and dotted lines in Fig. 1. We limit ourselves to  $0.6 < Q^2 < 6.0 \text{ GeV}^2$ , to match the  $Q^2$  range of precise polarization transfer data. We then use the difference between  $R_{L-T}$  and  $R_{Pol}$  to determine  $Y_{2\gamma}$ , using  $R_{Pol}$  as the approximate value for  $\tilde{G}_E/\tilde{G}_M$  in Eqs. 2

and 3. Note that if one instead uses the final value of  $\tilde{G}_E/\tilde{G}_M$  extracted from this analysis, the change is negligible. There is no way to extract the  $\varepsilon$ -dependence of  $Y_{2\gamma}$  because we only have a single value of  $R_{L-T}$  at each  $Q^2$  value, taken from the full  $\varepsilon$  range of the cross section data, and because the polarization transfer ratios have not been measured at different  $\varepsilon$  values. However, if the  $\varepsilon$ -dependence in the amplitudes is large enough to introduce a significant nonlinearity, then the reduced cross section as a function of  $\varepsilon$  would deviate from the linearity predicted in the one-photon exchange approximation. While current data is not precise enough to set tight limits on deviations from linearity, the existing data is all consistent with a linear dependence. Therefore, we assume that  $\tilde{G}_E$ ,  $\tilde{G}_M$ , and  $Y_{2\gamma}$  are independent of  $\varepsilon$ . Of course, given a model of the  $\varepsilon$ -dependence, or measurements of the nonlinearities in the two-photon exchange effects, we could incorporate this information on the  $\varepsilon$ -dependence into the fit to make an improved extraction.

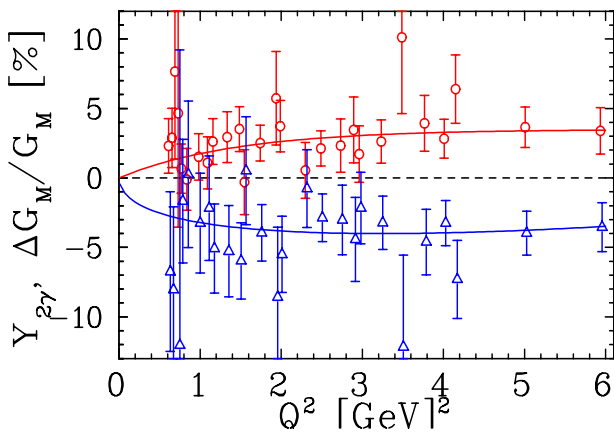


FIG. 2: (Color online) Extracted values of  $Y_{2\gamma}$  (red circles) and  $\Delta G_M/G_M$  (blue triangles), along with fits to the extracted amplitudes. The fits and parameterized uncertainties are given in the appendix.

Figure 2 shows the extracted values for  $Y_{2\gamma}$  as a function of  $Q^2$ , along with a fit to these extracted values, given in Eq. 5. Guichon and Vanderhaeghen [9] performed a similar extraction of  $Y_{2\gamma}$ , using fits to  $R_{Pol}$  and  $R_{L-T}$ . They extracted  $G_E/G_M$  from the polarization transfer data, correcting for the contribution of  $Y_{2\gamma}$  and assuming that  $\Delta G_E$  and  $\Delta G_M$  are negligible. However, while  $Y_{2\gamma}$  yields the *difference* between the two techniques, as well as the largest correction to the recoil polarization ratio, comparisons of positron-proton and electron-proton scattering demonstrate that  $\Delta G_M$  and  $\Delta G_E$  cannot be neglected [12]. The values of  $Y_{2\gamma}$  required to explain the discrepancy yield a 5–8% enhancement of the electron cross section at large  $\varepsilon$ . Because the dominant term of the two-photon correction changes sign for positron-proton scattering, this would imply a decrease in the positron-proton cross section, and a ra-

tio of positron to electron scattering of  $\lesssim 0.9$ . Data from Mar *et al.* [14] at large  $\varepsilon$  and  $Q^2$  yields an average ratio of  $1.017 \pm 0.024$ , well above this expectation.

One therefore needs additional input to constrain  $\Delta G_M$  and  $\Delta G_E$ . Precise comparisons of positron and electron scattering over a wide range in  $Q^2$  and  $\varepsilon$  would allow the extraction of these amplitudes, but the positron-proton scattering data above  $Q^2 = 1.3 \text{ GeV}^2$  is limited to small scattering angles, corresponding to  $\varepsilon > 0.7$ . Because of the very limited  $\varepsilon$  range, the positron data cannot be used to constrain the  $\varepsilon$ -dependence at large  $Q^2$  values. However, they still provide a useful constraint for the two-photon amplitudes. The positron data at large  $\varepsilon$  all indicate small two-photon contributions. To be consistent with this data, the contribution of  $Y_{2\gamma}$  to the cross section at large  $\varepsilon$  must be cancelled by the contributions of  $\Delta G_E$  and  $\Delta G_M$ . The change in the cross section due to  $\Delta G_E/G_E$  is suppressed with respect to the contribution from  $\Delta G_M/G_M$  by a factor of  $\varepsilon\rho^2/\tau$  (Eq. 4), which is below 0.15 for  $Q^2=2 \text{ GeV}^2$  and below 0.01 for  $Q^2 = 5.6 \text{ GeV}^2$ . Therefore, unless  $\Delta G_E/G_E$  is much larger than  $\Delta G_M/G_M$ , the  $Y_{2\gamma}$  contribution to the cross section at large  $\varepsilon$  must be cancelled almost entirely by  $\Delta G_M$ . Given the value of  $Y_{2\gamma}$ , we can determine  $\Delta G_M$  by requiring that the two-photon contribution to the cross section (Eq. 4) from  $Y_{2\gamma}$  at  $\varepsilon = 1$  be cancelled by the contribution from  $\Delta G_M$ :  $\Delta G_M/G_M = -(1 + \rho/\tau)Y_{2\gamma}$ . Figure 2 shows  $\Delta G_M/G_M$  as determined from the above procedure, as well as a fit to these extracted values (Eq. 6). Note that these amplitudes are a few percent of the Born amplitudes, larger than previously believed but still of order  $\alpha_{EM}$ .

The remaining two-photon amplitude,  $\Delta G_E$ , is more difficult to constrain, as it has a much smaller effect on the cross section. However, because both  $Y_{2\gamma}$  and  $\Delta G_E$  yield a correction to the cross section that is proportional to  $\varepsilon$ , the  $\varepsilon \rightarrow 0$  limit can be used to extract  $G_M$  with minimal uncertainty from  $Y_{2\gamma}$  and  $\Delta G_E$ . So the lack of information on  $\Delta G_E$  only affects extracted values of  $G_E/G_M$ . For this analysis, we take  $\Delta G_E$  to be zero, and use the larger of  $Y_{2\gamma}$  and  $\Delta G_M/G_M$  as an estimate of the uncertainty of  $\Delta G_E$ . This yields an additional uncertainty on the extracted value of  $G_E/G_M$  of approximately 3–4%, which is smaller than the typical experimental uncertainties from the polarization transfer data.

Having extracted the two-photon amplitudes, we can correct the polarization transfer measurements to yield the true form factor ratio,  $G_E/G_M$ . We use the fits to  $Y_{2\gamma}$  and  $\Delta G_M$  (Eqs. 5 and 6) to correct the polarization transfer measurements according to Eq. 2. This yields a correction of approximately 5% at low  $Q^2$  values, growing to 35% at  $Q^2 = 5.6 \text{ GeV}^2$ . The fractional uncertainty in these amplitudes is  $\sim 50\%$  for large  $Q^2$  values (above 3-4  $\text{GeV}^2$ ), and increases to 100% for low  $Q^2$  values ( $\approx 1 \text{ GeV}^2$ ). We parameterize the uncertainties in the extraction of  $Y_{2\gamma}$ ,  $\Delta G_M/G_M$ , and  $\Delta G_E/ge$

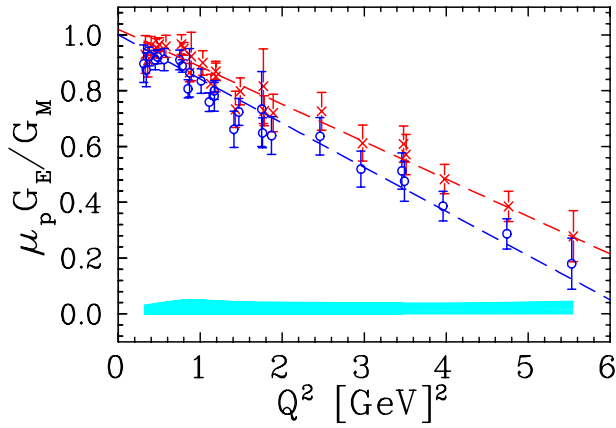


FIG. 3: (Color online) Polarization transfer measurements of  $\mu_p G_E/G_M$  as determined using the one photon exchange approximation (red  $\times$ ) and after applying the corrections based on the extraction of two-photon contributions as described in the text (blue circles). The error band at the bottom shows the uncertainties associated with the two-photon corrections. The corrected data are well fit by  $\mu_p G_E/G_M = 1 - 0.158Q^2$  (bottom dashed line).

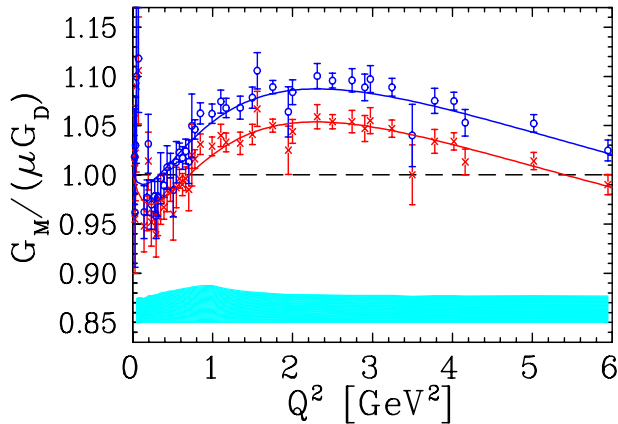


FIG. 4: (Color online) Rosenbluth extraction of  $G_M/\mu_p G_D$  as determined using the one photon exchange approximation (red  $\times$ ) and after applying the corrections based on the extraction of two-photon contributions as described in the text (blue circles). The error band at the bottom shows the uncertainties associated with the two-photon corrections. The data are compared to fits from Ref. [6], where the bottom curve (red) is the global analysis of the cross section data while the top curve (blue) includes both cross section and polarization transfer data, assuming an 6%, linear,  $\varepsilon$ -dependent correction to the cross section data.

(Eqs. 8, 9, and 10), and use this to determine the uncertainty in the two-photon corrections we apply to the polarization transfer ratio. The dominant uncertainties in the extraction of  $G_E/G_M$  are the experimental uncertainties in the polarization transfer measurement (typically 3–15%), the uncertainty in the extracted values of  $Y_{2\gamma}$  and  $\Delta G_M$  (4–12%), and the lack of knowledge of  $\Delta G_E$  (3–4%). Figures 3 and 4 show the uncorrected

and corrected values of  $G_E/G_M$  and  $G_M$  with the experimental uncertainties shown on the points and the uncertainties related to the two-photon corrections shown in the error bar on the bottom. A linear fit to the corrected data yields  $\mu_p G_E/G_M = 1 - 0.158Q^2$  (the uncorrected data yield  $\mu_p G_E/G_M = 1 - 0.135(Q^2 - 0.24)$  [3]).

Next, we can use the low  $\varepsilon$  cross sections, where  $Y_{2\gamma}$  and  $\Delta G_E$  have little effect (Eq.4), to extract  $\tilde{G}_M$ . We take the limit as  $\varepsilon \rightarrow 0$ , as in the usual L-T separation, and remove the two-photon contribution,  $\Delta G_M$ , as determined from the above analysis to yield the corrected value for  $G_M$ . The dominant uncertainties are the experimental cross section uncertainties (1–2% uncertainty in  $G_M$ ) and the uncertainty in  $\Delta G_M$  (1.5–3%). There is an additional uncertainty (0.5%), coming from the uncertainty in the large  $\varepsilon$  ratio of positron to electron cross section, which is only known to  $\sim 1\%$  from the existing positron data.

In extracting the two-photon amplitudes and the uncertainties in the corrected form factors we assumed that the two-photon amplitudes were independent of  $\varepsilon$ . Any  $\varepsilon$ -dependence must be small enough that it does not spoil the observed linearity of the reduced cross sections. However, it could still be large enough to yield a noticeable modification to the extracted value of  $G_M$ . If the amplitudes have nonlinearities at low  $\varepsilon$ , then the value of  $G_M$  will have an additional correction. Most of the available Rosenbluth separations at large  $Q^2$  are limited to  $\varepsilon \gtrsim 0.2$ , and therefore have to make a significant extrapolation. We can estimate the size of this uncertainty by examining measurements of the linearity of the reduced cross sections that have been performed as tests of the one-photon approximation.

A simple way to parameterize the limit on non-linearity is to fit the reduced cross sections to a quadratic rather than a linear equation,  $\sigma_r = P_0(1 + P_1\varepsilon + P_2\varepsilon^2)$ , and use the uncertainty on the  $\varepsilon^2$  coefficient,  $P_2$ , as an estimate of the possible nonlinear terms. The best linearity limits at high  $Q^2$  come from the SLAC NE11 experiment [15]. Their best measurement is at  $Q^2=2.5 \text{ GeV}^2$ , yields  $P_2 = 0.0 \pm 0.11$ . Similar limits ( $\delta P_2 = 0.12\text{--}0.2$ ) are set by data at lower  $Q^2$  [16]. In many cases, there are more data point at large  $\varepsilon$  values, increasing the uncertainty in the extrapolation to  $\varepsilon = 0$  beyond what one would estimate from the simple quadratic fit. The simple estimate of the allowed nonlinearities yields possible deviations of 3–4% in the extraction of the cross section at  $\varepsilon = 0$ , more for cases where the linearity measurements are not as precise or where there is a larger extrapolation to  $\varepsilon = 0$ . We include a 4% uncertainty on the cross section (2% uncertainty on  $G_M$ ) due to the possible error in the extrapolation to  $\varepsilon = 0$ .

To obtain the corrected form factors, we assumed that the discrepancy is fully explained by higher order radiative corrections, specifically two-photon exchange and Coulomb distortion, and had to make some assump-

tions about the  $\varepsilon$ -dependence. To the extent that these are valid assumptions (or good approximations), we obtain the correct two-photon amplitudes and can obtain the corrected form factors. One way to test these assumptions is to use the extracted amplitudes to predict the effect of these higher order terms in comparisons of positron- and electron-proton scattering. While we incorporate the  $\varepsilon = 1$  constraint of the positron measurements into the extraction, we have not included any other information on the  $\varepsilon$ -dependence. Unfortunately, data at smaller  $\varepsilon$  values is limited to low  $Q^2$ , where the uncertainties in the two-photon amplitudes are approaching 100%. While we have to extrapolate to lower  $Q^2$  to compare to the positron data, we do reproduce the trend observed in the positron data.

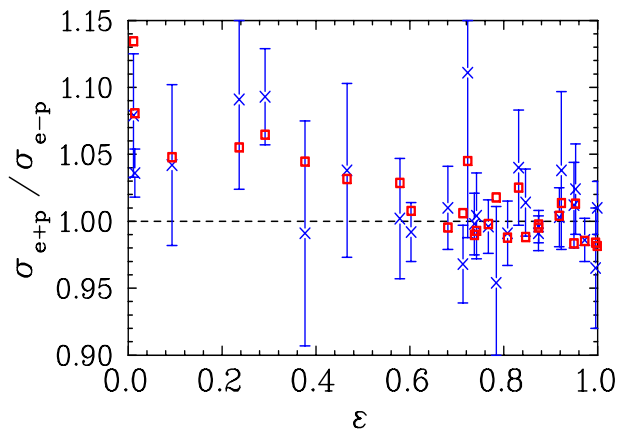


FIG. 5: (Color online) Ratio of positron-proton section to electron-proton cross sections (blue  $\times$ ), compared to the prediction from the extracted values of  $Y_{2\gamma}$  and  $\Delta G_M/G_M$ .

Figure 5 shows the ratio of positron to electron cross section as a function of  $\varepsilon$ , along with the predictions based on the fits to the two-photon amplitudes shown in Fig. 2. While the uncertainties are large, the data are in better agreement with the prediction from the two-photon amplitudes:  $\chi^2 = 18.2$  for 28 data points, compared with  $\chi^2 = 23.9$  if one assume  $\sigma_{e+p}/\sigma_{e-p} = 1$ , i.e. if the two-photon amplitudes are ignored. While the positron data could be included in the analysis to help constrain the two-photon amplitudes at low  $Q^2$ , it is clear that the current data would not significantly modify the values obtained using just the high- $\varepsilon$  constraint. Note that the two-photon prediction shown here does not yield exactly unity for  $\varepsilon \rightarrow 1$ . While the individual points for  $\Delta G_M/G_M$  from Fig. 2 are determined by requiring that this ratio approaches unity, the parameterizations of the two-photon amplitudes yield slight deviations.

## CONCLUSIONS

It is currently believed that the discrepancy comes from higher order corrections to the Born approximation. If this is true, and we assume a weak  $\varepsilon$ -dependence of the two-photon amplitudes, we can use the Rosenbluth, polarization transfer, and positron data to constrain the two-photon exchange (and other order  $\alpha_{EM}$ ) amplitudes. The large correction to Rosenbluth ratio allows extraction of  $Y_{2\gamma}$ , which then allows determination of the (smaller) correction to the polarization transfer. While the  $Y_{2\gamma}$  contribution to the polarization transfer ratio is as large as 35% for the largest  $Q^2$  value, the overall trend of a roughly linear decrease in  $\mu_p G_E/G_M$  with  $Q^2$  remains. Using the constraint from the two-photon (and multi-photon) exchange, positron cross section measurements at large  $\varepsilon$ , we also determine  $\Delta G_M$ . While we cannot directly constraint  $\Delta G_E$ , it has a relatively small effect on the extraction of the form factors, as long as it is not much larger than the other two-photon amplitudes.

Given these constraints on the two-photon amplitudes, we can correct  $G_E$  and  $G_M$  for two-photon exchange effects, with additional uncertainties associated with these corrections. For  $G_M$ , the uncertainty is dominated by possible  $\varepsilon$ -independence of the amplitudes, which can lead to deviations from the linear extrapolation to  $\varepsilon = 0$ . For  $G_E$  (and  $G_E/G_M$ ), the uncertainties are dominated by the large uncertainties in the  $R_{L-T}$  data, which limit the precision with which we can extract  $Y_{2\gamma}$  and  $\Delta G_M$ . We can use the two-photon amplitudes we extract to provide corrected values of  $G_E$  and  $G_M$ , but with additional uncertainties related to these correction, yielding final uncertainties that are 50-100% larger than the experimental uncertainties.

Additional positron data at low  $\varepsilon$  and moderate  $Q^2$  could be used to test the assumption that the discrepancy is fully explained by two-photon exchange, and give direct information on the  $\varepsilon$ -dependence. Such measurements are planned at Novosibirsk [17], and at Jefferson Lab [18]. Additional Rosenbluth measurements, utilizing the improved Rosenbluth technique of Ref. [5] can provide an improved extraction of  $R_{L-T}$  at moderate to large  $Q^2$  values, as well as better constraints on the linearity of the reduced cross section, and thus the  $\varepsilon$ -dependence of the two-photon amplitudes. Additional constraints on the  $\varepsilon$ -dependence of the two-photon amplitudes can be obtained from measurements of the  $\varepsilon$ -dependence of the polarization transfer [19]. With such additional measurements, the two-photon amplitudes can be constrained well enough that the proton form factors can be extracted with uncertainties from the two-photon corrections that are comparable to or smaller than the experimental uncertainties.



## APPENDIX: DETAILS OF THE TWO-PHOTON AMPLITUDE EXTRACTIONS

The following are the fits to the extracted values and uncertainties for  $Y_{2\gamma}$ ,  $\Delta G_M/G_M$ , and  $\Delta G_E/G_E$ :

$$Y_{2\gamma} = 0.035 \cdot (1 - \exp(-Q^2/1.45)) \quad (5)$$

$$\Delta G_M/G_M = (0.0124Q^2 - 0.0445\sqrt{Q^2})\% \quad (6)$$

$$\Delta G_E/G_E = 0 \quad (7)$$

$$\delta Y_{2\gamma} = (0.008 + 0.03 \exp(-Q^2/0.7) + 0.0015Q^2) \quad (8)$$

$$\frac{\delta(\Delta G_M/G_M)}{(\Delta G_M/G_M)} = \frac{\delta Y_{2\gamma}}{Y_{2\gamma}} \quad (9)$$

$$\delta(\Delta G_E/G_E) = \max(|Y_{2\gamma}|, |(\Delta G_M/G_M)|) \quad (10)$$

with  $Q^2$  in  $\text{GeV}^2$ , and with the relative uncertainty on the two-photon corrections limited to 100% at low  $Q^2$  values. At large  $Q^2$  values (above 3–4  $\text{GeV}^2$ ), the uncertainties in the two-photon amplitudes are  $\approx 40$ –50%. Below  $Q^2 = 1 \text{ GeV}^2$ , the parameterization of the uncertainty on  $Y_{2\gamma}$  becomes as large as the value itself, so the uncertainty is taken to be 100% in the analysis. Because  $\Delta G_M/G_M$  is derived directly from  $Y_{2\gamma}$ , the fractional uncertainty on  $\Delta G_M$  is identical to the fractional uncertainty on  $Y_{2\gamma}$ . Finally, because we do not have any data with which to constrain  $\Delta G_E$ , we assume that  $\Delta G_E = 0$ , and take the uncertainty to be the larger of the other two-photon amplitudes.

These fits and uncertainties are used to determine the correction to  $G_M$  as extracted from the cross section measurements, and the correction to  $G_E/G_M$  as extracted from the polarization transfer data, as shown in Fig. 3.

Table I gives the corrected values for  $G_M$ , determined by taking the uncorrected value of  $G_M$  (i.e.  $\tilde{G}_M$ ) from a global analysis of the Rosenbluth data [6] and correcting for the extracted value of  $\Delta G_M$  (Eq. 6). Additional two-photon uncertainties come from the uncertainty in  $\Delta G_M$  and a 2% uncertainty due to the possibility of non-linear terms that modify the extrapolation to  $\varepsilon = 0$ .

TABLE I: Extracted values of  $G_M$  after correcting for two-photon exchange effects. The form factor is given with respect to the dipole form,  $G_D = (1 + Q^2/0.71)^{-2}$ , with the experimental uncertainties listed first, and the additional uncertainties related to two-photon effects listed second.

$Q^2 [\text{GeV}^2]$	$G_M/(\mu_p G_D)$
0.005	0.751 $\pm$ 0.424 $\pm$ 0.016
0.011	0.981 $\pm$ 0.104 $\pm$ 0.021
0.015	1.000 $\pm$ 0.074 $\pm$ 0.021
0.018	0.973 $\pm$ 0.073 $\pm$ 0.021
0.022	1.018 $\pm$ 0.049 $\pm$ 0.022
0.027	0.962 $\pm$ 0.056 $\pm$ 0.021
0.034	1.030 $\pm$ 0.049 $\pm$ 0.022
0.045	1.100 $\pm$ 0.088 $\pm$ 0.024
0.072	1.118 $\pm$ 0.055 $\pm$ 0.025
0.141	0.962 $\pm$ 0.027 $\pm$ 0.025
0.179	0.977 $\pm$ 0.018 $\pm$ 0.026
0.195	1.031 $\pm$ 0.030 $\pm$ 0.027

0.234	0.970 $\pm$ 0.025 $\pm$ 0.027
0.273	0.978 $\pm$ 0.015 $\pm$ 0.028
0.292	0.958 $\pm$ 0.023 $\pm$ 0.028
0.312	0.977 $\pm$ 0.019 $\pm$ 0.029
0.350	0.996 $\pm$ 0.026 $\pm$ 0.030
0.389	0.989 $\pm$ 0.012 $\pm$ 0.030
0.428	1.007 $\pm$ 0.022 $\pm$ 0.031
0.473	1.009 $\pm$ 0.012 $\pm$ 0.032
0.507	0.984 $\pm$ 0.027 $\pm$ 0.032
0.545	1.013 $\pm$ 0.021 $\pm$ 0.033
0.584	1.023 $\pm$ 0.010 $\pm$ 0.034
0.622	1.017 $\pm$ 0.011 $\pm$ 0.034
0.663	1.024 $\pm$ 0.012 $\pm$ 0.035
0.701	1.013 $\pm$ 0.022 $\pm$ 0.035
0.740	1.050 $\pm$ 0.032 $\pm$ 0.036
0.778	1.046 $\pm$ 0.010 $\pm$ 0.036
0.846	1.062 $\pm$ 0.011 $\pm$ 0.037
0.992	1.062 $\pm$ 0.010 $\pm$ 0.038
1.102	1.074 $\pm$ 0.012 $\pm$ 0.035
1.168	1.068 $\pm$ 0.010 $\pm$ 0.034
1.344	1.068 $\pm$ 0.012 $\pm$ 0.032
1.496	1.078 $\pm$ 0.011 $\pm$ 0.031
1.557	1.106 $\pm$ 0.018 $\pm$ 0.031
1.751	1.089 $\pm$ 0.007 $\pm$ 0.029
1.947	1.064 $\pm$ 0.025 $\pm$ 0.028
2.000	1.084 $\pm$ 0.013 $\pm$ 0.028
2.308	1.100 $\pm$ 0.013 $\pm$ 0.028
2.499	1.096 $\pm$ 0.008 $\pm$ 0.028
2.743	1.096 $\pm$ 0.012 $\pm$ 0.028
2.904	1.089 $\pm$ 0.014 $\pm$ 0.027
2.972	1.097 $\pm$ 0.014 $\pm$ 0.028
3.243	1.089 $\pm$ 0.009 $\pm$ 0.027
3.497	1.040 $\pm$ 0.032 $\pm$ 0.027
3.777	1.075 $\pm$ 0.013 $\pm$ 0.027
4.018	1.075 $\pm$ 0.009 $\pm$ 0.027
4.160	1.053 $\pm$ 0.014 $\pm$ 0.027
5.017	1.052 $\pm$ 0.009 $\pm$ 0.027
5.945	1.025 $\pm$ 0.011 $\pm$ 0.027
7.037	0.989 $\pm$ 0.015 $\pm$ 0.026
9.121	0.943 $\pm$ 0.020 $\pm$ 0.023

TABLE II: Extracted form factor ratio  $G_E/G_M$  from polarization transfer experiments and corresponding value of  $G_E$  after applying the two-photon corrections as described in the text. In both cases, the experimental uncertainty is listed first, and the additional uncertainty related to the two-photon effects is listed second.

Ref.	$Q^2$ [ $\text{GeV}^2$ ]	$\mu_p G_E/G_M$ [corrected]	$G_E/G_D$ [corrected]
[20]	0.380	0.910 $\pm$ 0.053 $\pm$ 0.035	0.909 $\pm$ 0.055 $\pm$ 0.040
	0.500	0.969 $\pm$ 0.053 $\pm$ 0.043	0.979 $\pm$ 0.055 $\pm$ 0.048
[21]	0.373	0.961 $\pm$ 0.054 $\pm$ 0.035	0.959 $\pm$ 0.056 $\pm$ 0.040
	0.401	0.971 $\pm$ 0.053 $\pm$ 0.036	0.972 $\pm$ 0.055 $\pm$ 0.041
[1]	0.441	0.895 $\pm$ 0.051 $\pm$ 0.036	0.899 $\pm$ 0.053 $\pm$ 0.041
	0.490	0.921 $\pm$ 0.025 $\pm$ 0.039	0.930 $\pm$ 0.028 $\pm$ 0.044
	0.790	0.889 $\pm$ 0.023 $\pm$ 0.051	0.923 $\pm$ 0.026 $\pm$ 0.056
	1.180	0.800 $\pm$ 0.030 $\pm$ 0.047	0.854 $\pm$ 0.033 $\pm$ 0.053
	1.480	0.724 $\pm$ 0.048 $\pm$ 0.042	0.784 $\pm$ 0.052 $\pm$ 0.048
	1.770	0.649 $\pm$ 0.054 $\pm$ 0.040	0.708 $\pm$ 0.059 $\pm$ 0.046
	1.880	0.639 $\pm$ 0.068 $\pm$ 0.039	0.699 $\pm$ 0.075 $\pm$ 0.045
	2.470	0.637 $\pm$ 0.068 $\pm$ 0.039	0.699 $\pm$ 0.075 $\pm$ 0.045
	2.970	0.519 $\pm$ 0.064 $\pm$ 0.038	0.567 $\pm$ 0.070 $\pm$ 0.043

	3.470	0.512±0.065±0.039	0.555±0.071±0.044
[22]	0.320	0.897±0.067±0.030	0.891±0.068±0.035
	0.350	0.875±0.061±0.031	0.871±0.062±0.036
	0.390	0.923±0.033±0.034	0.923±0.036±0.039
	0.460	0.910±0.035±0.037	0.916±0.037±0.042
	0.570	0.912±0.040±0.041	0.928±0.042±0.046
	0.760	0.910±0.035±0.047	0.943±0.038±0.053
	0.860	0.807±0.033±0.048	0.843±0.036±0.054
	0.880	0.864±0.087±0.049	0.904±0.091±0.055
	1.020	0.835±0.044±0.051	0.883±0.047±0.057
	1.120	0.759±0.034±0.047	0.808±0.037±0.053
	1.180	0.781±0.055±0.047	0.834±0.059±0.053
	1.420	0.662±0.065±0.041	0.715±0.071±0.047
	1.760	0.734±0.134±0.042	0.801±0.146±0.049
[2]	3.500	0.475±0.072±0.038	0.515±0.078±0.043
	3.970	0.386±0.053±0.039	0.414±0.057±0.043
	4.750	0.287±0.054±0.041	0.302±0.057±0.044
	5.540	0.180±0.092±0.044	0.185±0.095±0.046

Given  $G_E/G_M$  and  $G_M$ , we can obtain  $G_E$ . However, we have extracted  $G_E/G_M$  from the polarization transfer measurements, and  $G_M$  from the cross section measurements, both corrected for the two-photon amplitudes. Because the uncertainty on  $G_M$  is much smaller, we extract  $G_E$  at the kinematics of the polarization transfer measurements, using the corrected values of  $G_E/G_M$  and a fit to the corrected values (and uncertainties) of  $G_M$ . Tables I and II give the corrected values for  $G_M$  and  $G_E$ , relative to the dipole form.

The corrected form factors are well described by the Polarization form factor fit to  $G_M$  from ref. [6] (top curve in Fig. 4), and  $\mu_p G_E/G_M = 1 - 0.158Q^2$  (bottom curve in Fig. 3). The fit for  $G_M$  from ref. [6] is nearly identical to the best fit to the corrected  $G_M$  data; the only noticeable difference is that it is slightly lower (up to 1%) for  $Q^2$  values of 2–4 GeV<sup>2</sup>. The fit by Brash, *et al.* [23], is 1.5–2.5% below the corrected values of  $G_M$  for  $Q^2 \gtrsim 1$  GeV<sup>2</sup>.

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